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EVALUATION OF MIXED-MODE STRESS INTENSITY FACTORS AND T-STRESS FOR 3-D SURFACE FLAWS

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ABSTRACT

Accurate fracture parameter evaluation is essential for predicting crack initiation and propagation in structural components containing surface flaws. In three-dimensional cracked bodies, stress intensity factors (SIFs) and T-stress govern crack-tip behavior and structural integrity under linear elastic conditions. This study presents a detailed finite element–based investigation of mixed-mode SIFs (K_1, K_2, K_3) and T-stress for representative three-dimensional surface cracks using ANSYS Mechanical.

Two benchmark configurations are analyzed: (i) a rectangular block containing a semicircular surface crack under tensile loading, and (ii) a tubular X-joint containing a warped semi-elliptical surface crack subjected to pressure loading. The interaction integral method (CINT) is employed to extract fracture parameters along the crack front. Structured SOLID186 elements are used in the crack-front region to accurately capture stress singularities.

Keywords: Fracture Mechanics, Stress Intensity Factor, T-Stress, Interaction Integral, Surface Crack, ANSYS Mechanical, Mixed-Mode Fracture

1. INTRODUCTION

Fracture mechanics provides a rational framework for assessing structural integrity in the presence of cracks and flaws. Unlike classical strength-of-materials approaches, fracture mechanics explicitly accounts for stress singularities that arise at crack tips. Even small surface cracks can significantly amplify local stresses, potentially leading to catastrophic failure.

In linear elastic fracture mechanics (LEFM), crack-tip severity is quantified by:

- Mode I stress intensity factor (K_1) – opening mode
- Mode II stress intensity factor (K_2) – in-plane shear
- Mode III stress intensity factor (K_3) – out-of-plane shear

In addition to singular terms, the non-singular T-stress plays a crucial role in crack-tip constraint and fracture stability. Accurate evaluation of both SIFs and T-stress is therefore essential for reliable fracture assessment.

Closed-form analytical solutions exist only for limited ideal geometries. Real engineering structures such as welded joints, pressure vessels, and tubular connections require numerical methods. The finite element method (FEM) has become the dominant tool for three-dimensional fracture analysis.

This work demonstrates systematic FEM-based evaluation of SIFs and T-stress using ANSYS Mechanical for both benchmark and complex crack geometries.

Literature Review

The assessment of fracture behavior in cracked structural components has been an active area of research for several decades, driven by the need to ensure the safety and reliability of engineering systems operating under demanding service conditions. Early developments in fracture mechanics focused primarily on idealized two-dimensional crack configurations; however, increasing computational capabilities and experimental observations have progressively shifted attention toward three-dimensional surface and embedded cracks in finite geometries.

Surface Cracks in Finite Plates

One of the most influential and widely cited contributions in the field of surface crack analysis is the work of **Newman and Raju**. They developed comprehensive analytical and semi-empirical formulations for stress intensity factors associated with semi-elliptical surface cracks and corner cracks in finite-thickness plates subjected to various loading conditions, including uniform tension and bending.

Relevance to the Present Work

The present study builds directly upon the extensive body of literature discussed above by employing **ANSYS Mechanical's specialized fracture mechanics capabilities**, including **Semi-Elliptical Crack** and **Pre-Meshed Crack** objects, in conjunction with the **interaction integral–**

based CINT method. By analyzing both a rectangular solid with a surface crack and a tubular X-joint with a warped surface flaw, the work bridges the gap between classical benchmark problems and complex real-world applications. The numerical results are systematically compared with established literature data to validate the modeling approach and to demonstrate the accuracy and robustness of the finite element–based fracture analysis.

2.7 Mesh Sensitivity and Crack-Tip Modeling Techniques

Accurate evaluation of stress intensity factors using finite element analysis strongly depends on the representation of the crack tip and the surrounding mesh. Several researchers have investigated mesh sensitivity and crack-tip element formulations to improve numerical accuracy. Early studies demonstrated that conventional finite elements are unable to capture the $r^{-1/2}$stress singularity at the crack tip unless extremely fine meshes are used, leading to high computational cost.

2. THEORETICAL BACKGROUND

2.1 Crack-Tip Stress Field

In LEFM, the near-tip stress field is expressed as:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + T\delta_{i1}\delta_{j1} + O(\sqrt{r})$$

Where:

- K= stress intensity factor
- r= radial distance from crack tip
- T= non-singular stress term
- $f_{ij}(\theta)$= angular stress functions

The first term represents the singular stress field, while the second term corresponds to T-stress.

2.2 Interaction Integral Method

The interaction integral method extends the classical J-integral to separate mixed-mode SIF components in three dimensions.

Advantages:

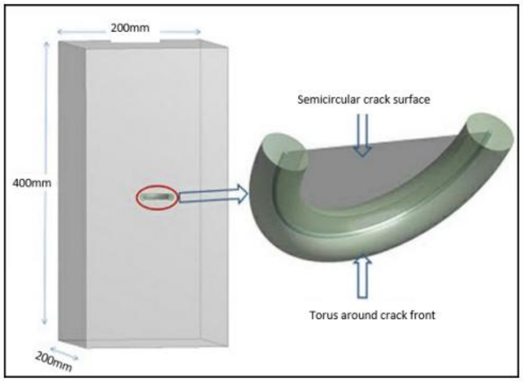
- Path-independent
- Less mesh-sensitive
- Direct extraction of K_1, K_2, K_3
- Capable of evaluating T-stress

This method is implemented in ANSYS Mechanical via the CINT command.

3. GEOMETRY CONFIGURATIONS

3.1 Rectangular Block with Semicircular Surface Crack

The rectangular block geometry is imported into ANSYS Workbench from CAD models prepared in **SpaceClaim and DesignModeler**, ensuring dimensional accuracy and compatibility with the finite element meshing tools. The block dimensions are chosen to be sufficiently large relative to the crack size so that boundary effects do not dominate the crack-tip stress field, thereby approximating conditions assumed in reference solutions.



- Crack radius: 20 mm
- Surface-breaking semicircular crack
- Tensile pressure loading
- Primarily Mode I fracture

This configuration serves as a benchmark problem.

3.2 Tubular X-Joint with Warped Surface Crack

- Two intersecting cylindrical members
- Semi-elliptical warped crack at weld toe
- Pressure loading on smaller tube
- Mixed-mode fracture behavior

Due to curvature and weld geometry, the crack front is non-planar.

4. MATERIAL MODEL

Both models use linear elastic isotropic structural steel:

Property	**Value**
Young’s Modulus	210 GPa
Poisson’s Ratio	0.3

Assumptions:

- Linear elasticity
- Small-scale yielding
- Homogeneous material

These conditions satisfy LEFM requirements.

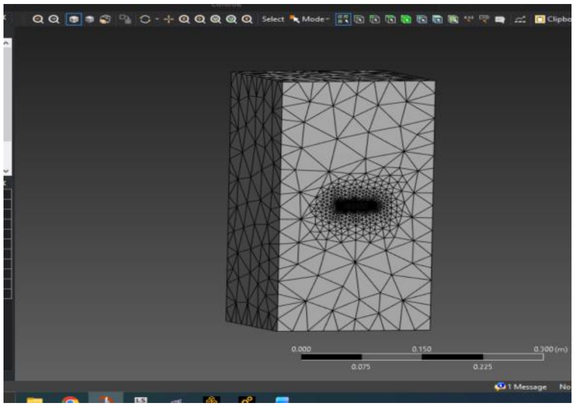
5. MESH STRATEGY

In summary, the mesh methods employed in this work are specifically tailored for three-dimensional fracture mechanics analysis. Structured SOLID186 elements are used in the crack-front region to accurately capture stress singularities and support interaction integral evaluation, while SOLID187 elements are used in the bulk regions to reduce computational cost. The meshing strategy ensures reliable computation of stress intensity factors and T-stress for both benchmark and complex welded joint configurations.

5.1 Crack-Front Elements

- SOLID186 quadratic elements near crack front
- Structured toroidal region
- Multiple interaction integral contours
- Refined crack-front discretization

Outside crack region: SOLID187 tetrahedral elements.



Mesh

5.2 Mesh Convergence

Validation performed by:

- Checking contour independence
- Comparing intermediate contours
- Verifying smooth SIF distribution

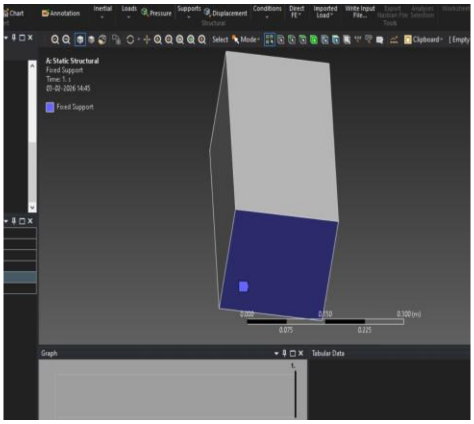
6. BOUNDARY CONDITIONS

6.1 Rectangular Block

- Fixed support on one face
- Uniform tensile pressure on opposite face

This generates dominant Mode I loading.

From a theoretical standpoint, the fixed support establishes a **reference boundary** that prevents rigid-body translation of the model and ensures that externally applied loads result in internal stresses rather than global motion. This type of constraint is commonly used in benchmark fracture problems and allows direct comparison with analytical and numerical reference solutions available in the literature.



Fixed support

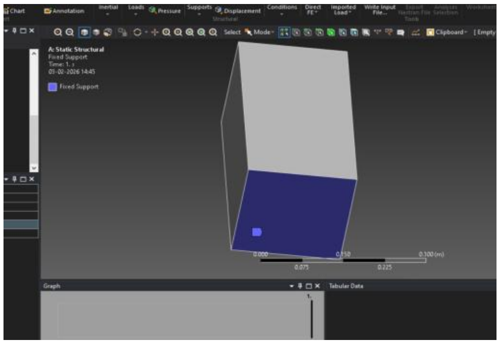
6.2 X-Joint Pipe

- Two symmetry planes
- Minimal stabilization constraint
- Pressure loading on smaller tube

This produces mixed-mode fracture conditions.

7. RESULTS AND DISCUSSION

7.1 Rectangular Block



Fixed support

6.1.2 Applied Loading

On the face opposite to the fixed support, a **uniform nodal pressure load of −10 × 10³ MPa** is applied. The negative sign indicates that the pressure acts inward, producing a tensile stress state within the rectangular block when viewed in the global coordinate system. This loading configuration generates a nearly uniform far-field tensile stress, which is a classical loading condition for surface-crack fracture studies.

Applied Loading

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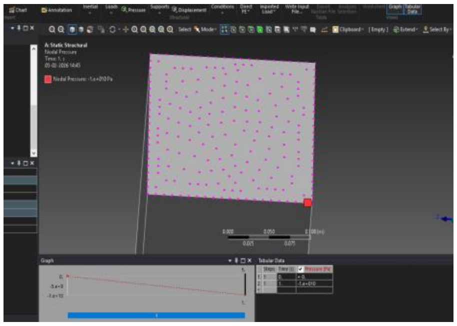
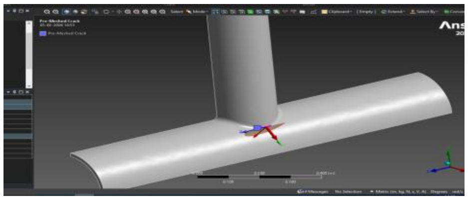


Nodal Pressure



Pre meshed crack

7.1.1 Deformation and Stress

- Maximum deformation ≈ 0.011 m
- Strong stress concentration at crack tip

Stress field follows expected singular pattern.

7.1.2 Mode I Stress Intensity Factor

Normalized:

$$\frac{K_1}{\sigma\sqrt{\pi a}}$$

Observations:

- Minimum near free surface
- Maximum at deepest crack point
- Symmetric distribution

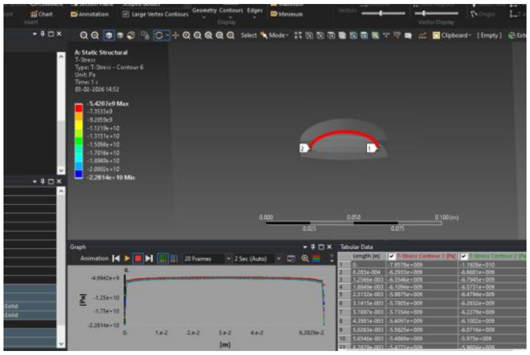
Agreement with Newman–Raju solutions confirms model accuracy.

7.1.3 T-Stress Distribution

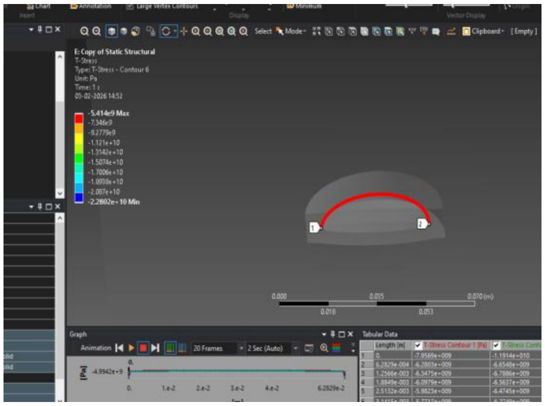
Normalized T-stress varies smoothly:

- Lower near free surface
- Higher toward interior

Indicates increased crack-tip constraint away from free surface.



T stress



T stress stainless steel

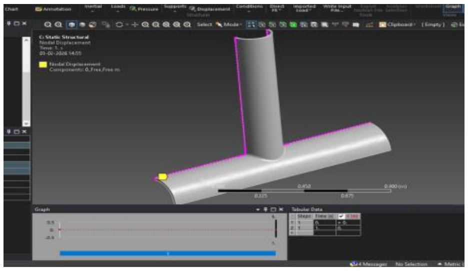
7.2 X-Joint Pipe

6.2.1 Use of Symmetry

The X-joint pipe geometry and loading conditions are symmetric with respect to two orthogonal planes. To reduce computational cost while maintaining accuracy, only **one quarter of the full structure** is modeled. Two planar symmetry boundary conditions are applied on the corresponding cut faces.

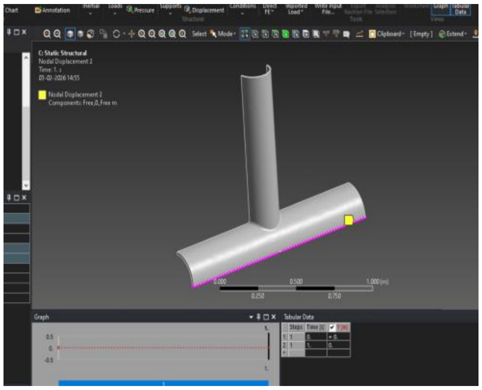
On each symmetry plane:

- The displacement component normal to the plane is constrained.
- Tangential displacement components are left unconstrained.



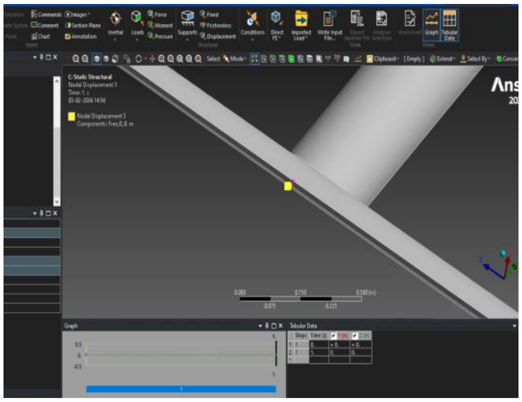
C: Static Structural
Nodal Displacement
Time: 1. s
01-02-2026 14:55
Nodal Displacement
Components: 0.,Free,Free m

Nodal displacement



C: Static Structural
Nodal Displacement 2
Time: 1. s
01-02-2026 14:55
Nodal Displacement 2
Components: Free,0.,Free m

Nodal displacement 2



C: Static Structural
Nodal Displacement 3
Time: 1. s
01-02-2026 14:56
Nodal Displacement 3
Components: Free,0.,0. m

Nodal displacement 3

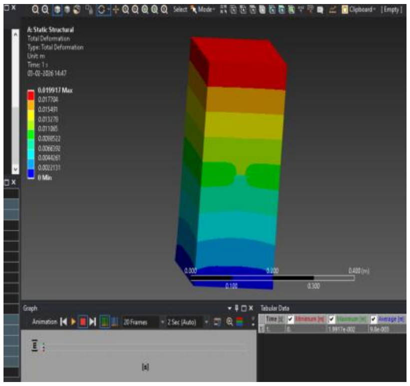
7.2.1 Global Stress Distribution

High stresses localized near weld toe and crack front.

Global Deformation Behavior

The total deformation contours reveal that the rectangular block undergoes smooth, continuous elastic deformation under the applied tensile pressure. As expected, the **maximum displacement occurs at the loaded face**, which is located farthest from the fixed support. The displacement magnitude gradually decreases toward the constrained face, reflecting the elastic load transfer through the body.

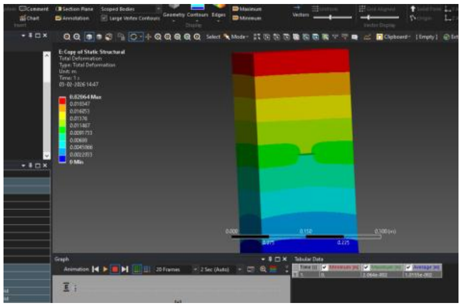
Importantly, the deformation is symmetric with respect to the mid-plane of the crack, further confirming that the geometry, loading, and boundary conditions are correctly implemented.



A: Static Structural
Total Deformation
Type: Total Deformation
Unit: m
Time: 1 s
01-02-2026 14:47

0.019917 Max
0.017704
0.015491
0.013278
0.011065
0.0088522
0.0066392
0.0044261
0.0022131
0 Min

Deformation



Stainless steel Deformation

7.2.2 Mixed-Mode SIFs

Computed results (Structural Steel):

Parameter	Value
K_1	540 MPa√m
K_2	92 MPa√m
K_3	16 MPa√m
T-stress	1600 MPa

Observations:

- Mode I dominant
- Significant Mode II contribution
- Minor Mode III
- Strong constraint effect reflected in T-stress

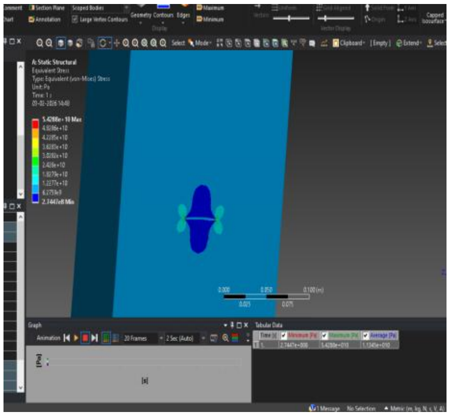
Good agreement with Rhee & Salama benchmark data.

7.2.3 Effect of Crack Warping

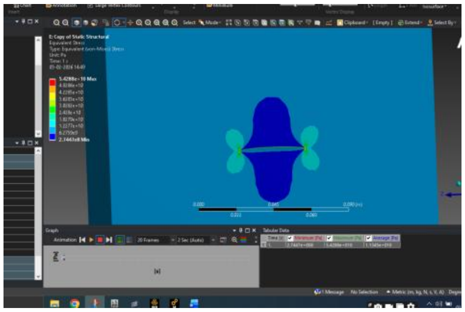
Warped geometry causes:

- Non-uniform mode mixity
- Variation in crack-tip constraint
- Endpoint sensitivity to mesh refinement

The observed stress concentration confirms that the mesh refinement and crack-front element topology are adequate to capture the near-tip stress gradients required for accurate SIF evaluation.



Equivalent stress



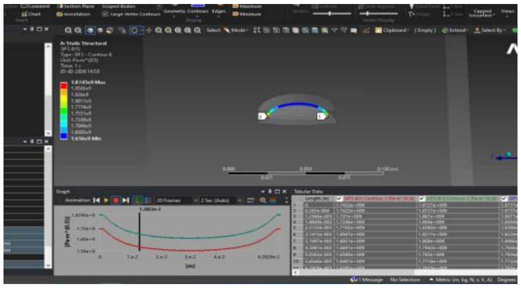
Stainless steel Equivalent stress

Mode I Stress Intensity Factor Distribution

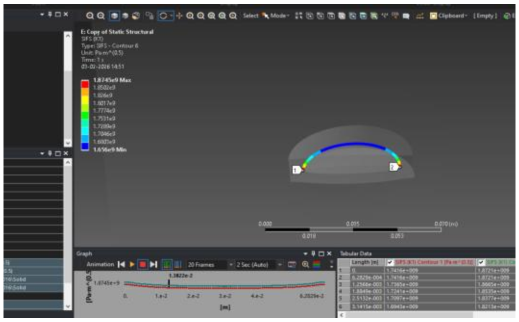
The mode I stress intensity factor K_I is computed along the crack front using the interaction integral method. To facilitate comparison with benchmark solutions, the results are normalized using the expression:

$$\frac{K_I}{\sigma\sqrt{\pi a}}$$

This trend is physically expected, as the crack experiences greater constraint away from the free surface, leading to higher stress intensity.



Stress intensity factor



Stress intensity factor stainless steel

8. COMPARATIVE ANALYSIS

Model	Dominant Mode	Complexity	Validation Level
Rectangular Block	Mode I	Simple	Excellent
X-Joint Pipe	Mixed Mode	High	Very Good

Key Findings:

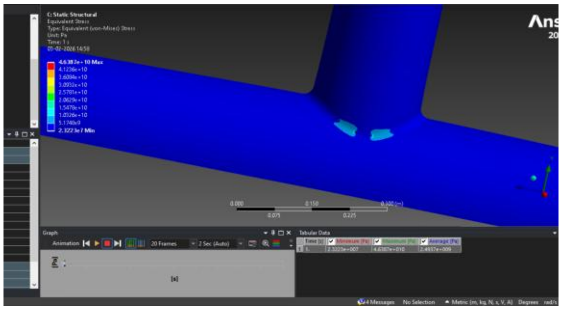
1. Interaction integral accurately separates mixed modes.
2. SOLID186 elements critical for crack-tip accuracy.
3. T-stress essential for constraint evaluation.
4. Warped crack fronts require refined mesh.
5. Benchmark validation confirms numerical robustness.

T-Stress in the X-Joint Pipe

The T-stress distribution along the warped crack front is also evaluated and presented as a function of normalized crack-front position. The results show significant variation in T-stress magnitude and sign along the crack front, reflecting:

- Changes in local constraint conditions,
- Interaction between global bending and local crack-tip stresses,
- Effects of weld geometry and curvature.

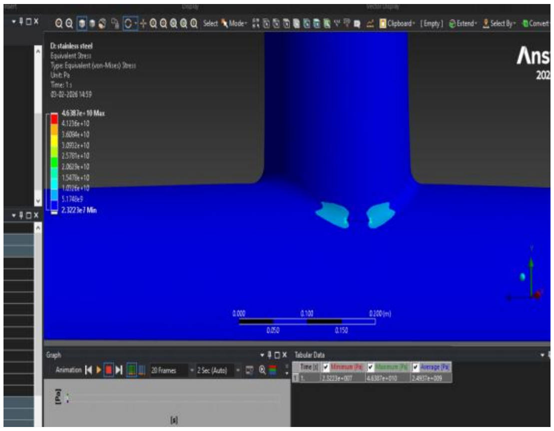
These findings highlight the importance of considering T-stress in addition to SIFs when assessing fracture behavior in complex three-dimensional structures.


C: Static Structural
Equivalent Stress
Type: Equivalent (von-Mises) Stress
Unit: Pa
Time: 1 s
03-02-2026 14:58

Tabular Data

	Time [s]	☑ Minimum [Pa]	☑ Maximum [Pa]	☑ Average [Pa]
1	1.	2.3223e+007	4.6387e+010	2.4937e+009

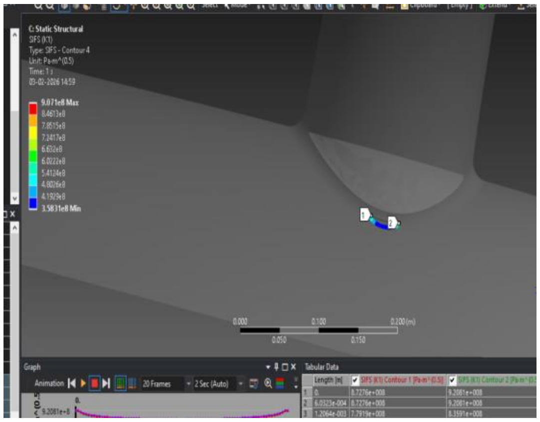
Equivalent stress


D: stainless steel
Equivalent Stress
Type: Equivalent (von-Mises) Stress
Unit: Pa
Time: 1 s
03-02-2026 14:59

Tabular Data

	Time [s]	☑ Minimum [Pa]	☑ Maximum [Pa]	☑ Average [Pa]
1	1.	2.3223e+007	4.6387e+010	2.4937e+009

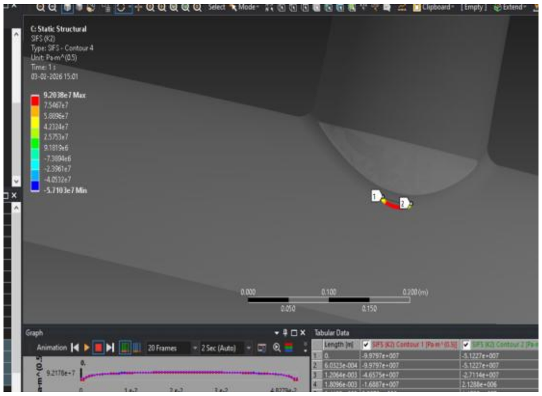
Equivalent stress stainless steel


C: Static Structural
SIFS (K1)
Type: SIFS - Contour 4
Unit: Pa·m^(0.5)
Time: 1 s
03-02-2026 14:59

Tabular Data

	Length [m]	☑ SIFS (K1) Contour 1 [Pa·m^(0.5)]	☑ SIFS (K1) Contour 2 [Pa·m^(0.5
1	0.	8.7276e+008	9.2081e+008
2	6.0323e-004	8.7276e+008	9.2081e+008
3	1.2064e-003	7.7919e+008	8.3591e+008

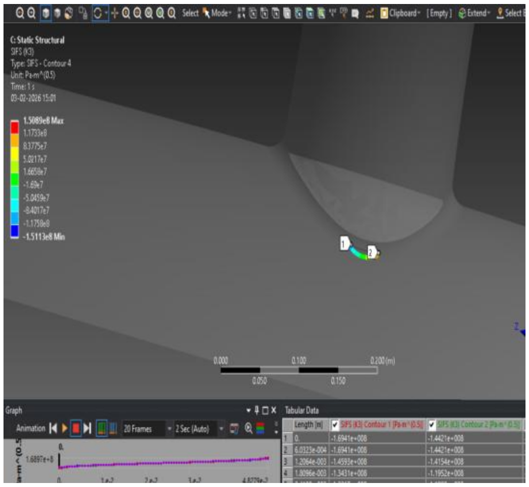
SIFS K1


C: Static Structural
SIFS (K2)
Type: SIFS - Contour 4
Unit: Pa·m^(0.5)
Time: 1 s
03-02-2026 15:01

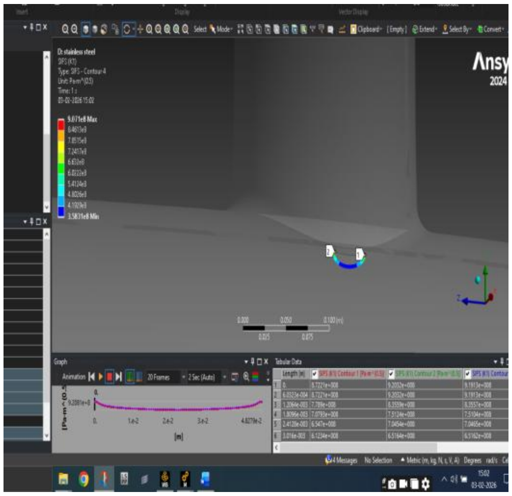
Tabular Data

	Length [m]	☑ SIFS (K2) Contour 1 [Pa·m^(0.5)]	☑ SIFS (K2) Contour 2 [Pa·m
1	0.	-9.9797e+007	-5.1227e+007
2	6.0323e-004	-9.9797e+007	-5.1227e+007
3	1.2064e-003	-4.6575e+007	-2.7114e+007
4	1.8096e-003	-1.6887e+007	2.1288e+006

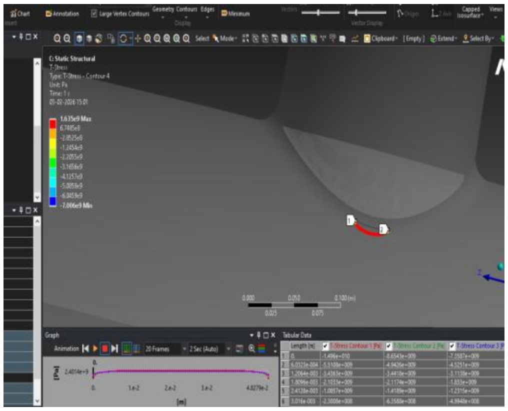
SIFS K2


C: Static Structural
SIFS (K3)
Type: SIFS - Contour 4
Unit: Pa·m^(0.5)
Time: 1 s
03-02-2026 15:01

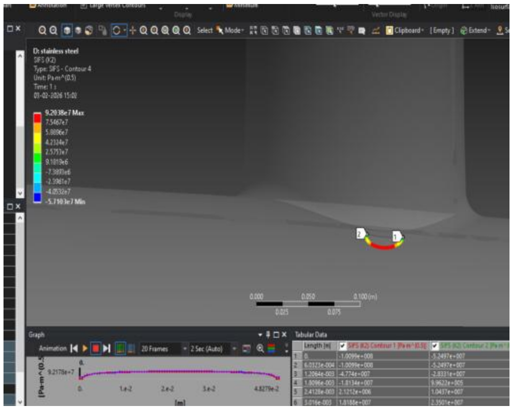
SIFS K3


D: stainless steel
SIFS (K1)
Type: SIFS - Contour 4
Unit: Pa·m^(0.5)
Time: 1 s
03-02-2026 15:02

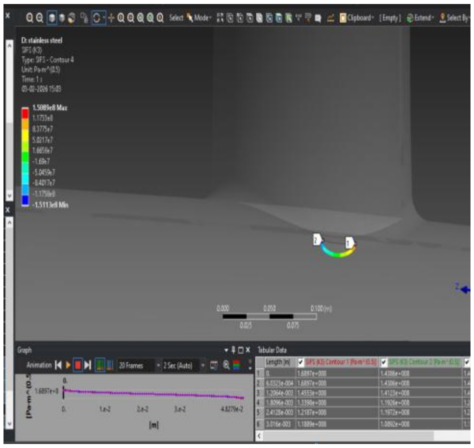
SIFS K1 SS


C: Static Structural
T-Stress
Type: T-Stress - Contour 4
Unit: Pa
Time: 1 s
03-02-2026 15:01

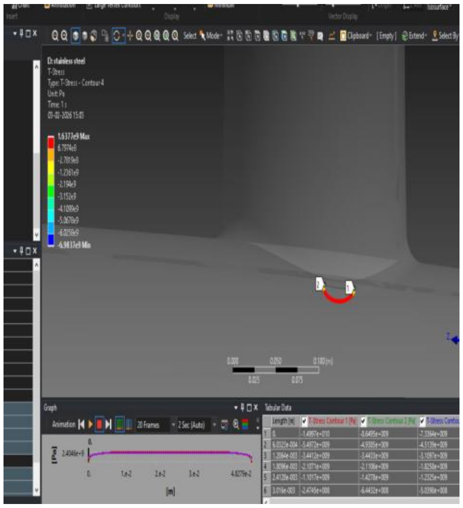
T stress


D: stainless steel
SIFS (K2)
Type: SIFS - Contour 4
Unit: Pa·m^(0.5)
Time: 1 s
03-02-2026 15:02

SIFS K2 SS


D: stainless steel
SIFS (K3)
Type: SIFS - Contour 4
Unit: Pa·m^(0.5)
Time: 1 s
03-02-2026 15:03

SIFS K3 T Stress SS



10. CONCLUSION

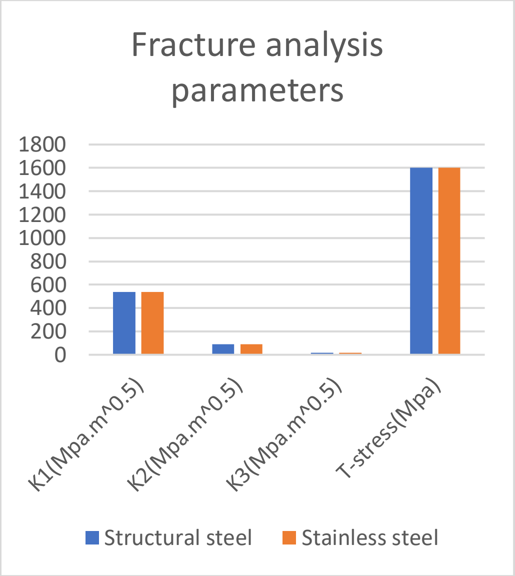
This study demonstrates robust finite element–based evaluation of mixed-mode stress intensity factors and T-stress for three-dimensional surface flaws.

Major conclusions:

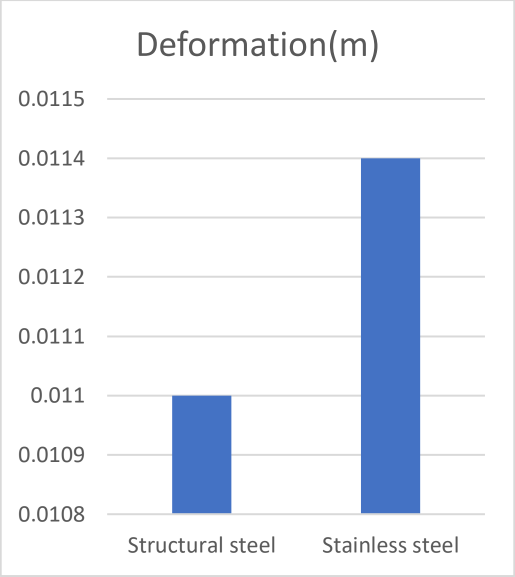
1. Numerical results match established analytical and FE benchmarks.
2. Interaction integral method provides accurate mixed-mode decomposition.
3. T-stress significantly influences crack constraint behavior.
4. Structured crack-front meshing is essential.
5. Methodology applicable to complex engineering structures.

The presented workflow provides a validated and practical framework for advanced three-dimensional fracture mechanics analysis.

S.No	Material	K1(Mpa.m^0.5)	K2(Mpa.m^0.5)	K3(Mpa.m^0.5)	T-stress(Mpa)
1	Structural steel	540	92	16	1600
2	Stainless steel	540	92	16	1600



S.No	Material	Deformation(m)
1	Structural steel	0.011
2	Stainless steel	0.0114



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